In this chapter we consider two interacting species, each with non-overlapping generations. Suppose they affect each other’s population dynamics.

**Discrete-time predator-prey model:** Consider the interactions between predators P and prey N of the form

 

 

Here, is the net linear birth rate of prey, the function  represents the inﬂuence of the predator on the birth rate and the function represents the eﬃciency with which the predator searches for the prey.

We first consider the following simple model where the predators search over a constant area, and have an unlimited capacity for consuming prey:

 

 

where  represents the strength of the predation eﬀect.

Let  and  are the equilibrium or steady states of (3) and (4). Then from (3) and (4), we have

, 

,  

From the first of (5), we get







From the second of (5), we get









Thus the positive steady state populations are

,  

For linear stability of equilibrium we put

, , ,  

in (3) and (4). Thus we have

 

and  

For the steady state , from (8), we have



 

For the steady state , from (9), we have



 

The equations (10) and (11) will be linear if







Using the value of  in (10) and (11), we get

 

 

If , then the steady state is stable since  as . If , then the steady is unstable. In this case the positive steady state (6) exists.

For the nonzero steady state, from (8), we get









For linear system this gives



 

Again for the nonzero steady state, from (9), we get











For linear system this gives







 

Thus, for positive steady state we have the linear system of equations

 

For getting a single equation of , we iterate the first equation and then use the second equation of (16). That is,







 

We now seek solutions in the form



 and 

The equation (17) becomes







 

Let the solutions are  and . Then

 

Thus,  

Where , are arbitrary constants.

In a similar way we then get  as

 

Where , are arbitrary constants.

The stability of steady state  is obtained by the magnitude of  and . If  or  then and become unbounded as  and hence  is unstable. If  then from (19) we see that



So  and  are complex, and one is the conjugate of others. The product of the roots is

, for all 



Thus the solutions  from (20) and (21) become unbounded as . So the positive equilibrium in (6) is unstable.